Policy Implications of Solutions of Dynamic Optimal Production Problems for Disinflationary Economic Policies

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Abstract

The speed of the application of current economic policies that are being followed in Turkey is a very important question. This paper uses the results of the solutions of optimal control theoretic inventory and production problems for companies to offer opinions on this question under various assumptions on interest rates. The most important result of the paper is that the rate of decline of interest rates is as important as the absolute level of interest rates for investors to start investing in inventories (consequently in machinery and other production infrastructure) which, in turn, implies growth of the economy.

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Objective

The current economic program applied by the Government aims, in the short run, to reverse the debt trap,rehabilitate the banking sector,end or limit the the need for short term external borrowing, and to restore the confidence in the markets by free exchange rate regime, monetisation of domestic debt, large external support (IMF and the World Bank), strict fiscal measures, and accelaration of structural reforms and privatization.

However, in the mid-long term, the aims of the economic program are to have sustainable export-led growth of the economy, decrease the role of the Government in the economy, and to improve the income distribution by prudent and transparent public finance policies, improving the efficiency of the markets by rendering the markets more competitive, proper regulation of the financial system, and fight against corruption.

From a strictly financial point of view, one of the principal aims of all of these policies is to reduce inflation, thus reduce interest rates, and, therefore, to induce growth of the economy and by doing so to reduce unemployment.

The speed of application of these policies, however, remains an interesting problem to investigate. A fast application may result in unwanted results to disturb the social peace. A slow application, on the other hand, may render these policies ineffective.

The objective of this paper is to use the results of the optimal production policies of manufacturing companies under various interest rate scenarios (especially in disinflationary periods) to offer opinions on the speed of application of these economic policies.

Model

The problem of optimal production and inventory scheduling has been studied extensively [see 1, 2, 6, 12, 13] in many books and articles.

In the model to be developed, following notations are used:

$u(t)$: Production rate at time t

$I(t)$: Inventory level at time t

$r(t)$: Demand rate at time t

h : Inventory cost per unit of inventory, a constant.

c (u) : Production cost, with C' > 0, C" > 0 implying production with increasing marginal costs.

The problem of the manufacturer, therefore, is to minimize the sum of cost of production and the inventory holding cost over a certain time horizon.

$$\text{Minimize} \int_0^{\infty} [\, c(u) + h.I \,]\, dt \tag{1}$$

by proper choice of u (t) . However, the inventory level, I (t), changes over time according to:

$$\frac{d\,I(t)}{d\,(t)} = u(t) - r(t) \tag{2}$$

i .e. inventory level increases by production, decreases by sales.

In addition, it should be noted that the production rate and the inventory level can not be negative:

$$u(t) \geq o \tag{3}$$

$$I(t) \geq o \tag{4}$$

It is assumed that the inventory level at the beginning of the planning period (t=0) is positive:

$$I(o) = I_0 > o \tag{5}$$

Then, the task is to solve the problem (1) with constraints in equations in (2-5). This problem is solved in [14]. The solution consists of regions where:

$$I(t) = 0 \tag{6}$$

and

$$I(t) > 0 \tag{7}$$

It is shown that in regions where I (t) = 0, u (t) = r (t), that is the optimal production is just equal to demand. It is shown also that in these regions, it is necessary to have:

$$C''(r(t)) \cdot \frac{dr}{dt} \leq h \tag{8}$$

for I (t) = 0 to be optimal.

Therefore, optimal production schedule can be constructed by determining where equation (8) is violated ($I > 0$) and where $I (t) = 0$

It should be noticed from equation (8) that as long as demand in falling, i. e. $dr / dt \leq 0$, equation (8) will not be violated and therefore $u (t) = r (t)$ will remain to be optimal. It is only when dr / dt is positive and high enough to violate equation (8), will $I(t) > 0$, i. e. $u (t) > r (t)$. The policy implication of this is that the producers will increase inventory only when the rate of increase of demand is high enough. Therefore, in disinflationary environments (where demand is falling), it is not optimal to expect investment in inventories or machineries by producers until the demand begins to increase at a high enough rate.

In the analysis above, the firm's objective was to minimize the total cost of production and inventory cost as expressed in equation (1). However, this equation does not take into account the time value of money (i.e. the interest rates). In the following section, the problem of minimizing the present value of production and holding costs will be addressed.

Mathematical issues such as existence, sufficiency, and steady-state conditions are not analyzed since the primary objective is to concentrate on the policy implications of the solution of the problems. A very good exposition of these issues can be found in [11].

Model With Constant Time Value of Money (m)

In this case, the problem becomes.

$$\text{Minimize.} \int_0^\infty e^{-mt} \left[c (u) + h.I \right] dt$$

subject to equation (2), (3), (4), and (5), where. The introduction of the concept of present value, i. e. m is the constant interest rate the interest rate(apart from the fact that it is theoretically more correct form of financial anaysis) allows us to determine the effect of interest rates on optimal investment behaviour of a manufacturing company.

The Hamiltonian for this problem is [3,4,10]

$$H = e^{-mt} \left[c (u) + h.I \right] + \rho (u-r) + \mu (r-u) \tag{9}$$

The necessary condition for optimality are:

(on subarc where $I (t) = 0$, and $u(t)=r(t)$ for $0 < t_1 \leq t \leq t_2 < \infty$) :

$$\partial H / \partial u = e^{-mt} \; c'(u) + \rho - \mu = 0 \tag{10}$$

$$\rho = -\frac{\partial_H}{\partial_I} = -h.\, e^{-mt} \tag{11}$$

$$\mu\,(t^-_2) = 0 \;\; [\text{ see } 4] \tag{12}$$

$$\dot{\mu} \le 0 \text{ for } t_1 \le t \le t_2 \tag{13}$$

As developed in equation 10,

On subarc where I (t) = 0 (i. e. u (t) = r (t)). Then equation (10) leads:

$$e^{-mt} c'(u) + \rho = \mu \tag{14}$$

or,

$$e^{-mt} \; c'(r) + \rho = \mu \tag{15}$$

since u (t) = r (t)

Then equation (13), (11) and (15) leads

$$\dot{\mu} = -h.\, e^{-mt} + c''.\; dr \frac{e^{-mt}}{dt} - c'\, m\, e^{-mt} \le 0$$

or,

$$c'' \cdot \frac{dr}{dt} \le h + c'\, m \tag{16}$$

which is very similar to equation (8).

As in previous case, it is optimal to keep I (t) = 0 as long equation (16) is not violated. Otherwise I > 0.

The implications of equation (16) are that:

. As long an demand is falling, i. e. dr. / dt < 0, it is optimal to keep I = 0

. Even if demand stops falling, i. e. dr / dt = 0, equation (16) will remain valid since c' > 0, hence it is still optimal to keep I = 0

. Rate of increase of demand has to be high enough so that equation (16) is violated for producers to start accumulating inventory, i. e. I > 0. In this case producers will have to wait longer than they did in the first case(see equation 8) to start production to build inventories since c'. m > 0 and it will longer time to violate

$$c'' \cdot \frac{dr}{dt} \leq h + c'.m$$

than to violate

$$c'' \cdot \frac{dr}{dt} \leq h \qquad (\text{ Equation } 8 \text{ })$$

. The introduction of interest rate into the analysis has the effect of postponing investment longer than the case where the interest rate was ignored. It is also clear that higher the the interest rate longer will be the period of postponement for investment.

In a country like Turkey where m is high, producers will start to build inventories later than they would if interest rates were lower. A different statement of this fact is that it will take longer to implement disinflationary policies in countries where interest rates are higher than in countries where the interest rates are lower. This, however is not a surprising result.

In the next section it will be assumed that interest rate is time dependent and is falling, i. e.

$$m = m(t) \text{ and } m^{l} < 0 \tag{17}$$

which is the case in Turkey.

In this case, the discount factor becomes $e^{-m(t).t}$ where m(t) is a time dependent function instead of a constant. This causes no change in the procedures to solve the problem.

The Model with Time Value of Money Falling

The necessary conditions for this case is exactly as in equations 10-13 except

$$m = m(t) \text{ and } m^{l} < 0$$

From equation (10), we still have (on I = 0 arc)

$$e^{-m(t).t} \quad c'(u) + \rho - \mu = 0 \tag{18}$$

$$\rho = -h.\, e^{-m(t).t} \tag{19}$$

$$\mu(t_2^{-}) = 0 \tag{20}$$

$$\dot{\mu} \leq 0 \quad \text{on} \tag{21}$$

$t_1 \leq t \leq t_2$ where I= 0 and therefore u(t) = r(t)

From equation (18) and u(t) = r(t), we have:

$$e^{-m(t).t} \quad c'(r) + \rho = \mu \tag{22}$$

From equations (22), (21), (19); it is necessary to have

$$\dot{\mu} = -h.\, e^{-m(t).t} + c'' \cdot \frac{dr}{dt} e^{-m(t).t} + c'(r)\,[\,-m^{l}\,t - m\,]\, e^{-m(t).t} \leq 0$$

or

$$c'' \cdot \frac{dr}{dt} \leq c'(m(t) + t\,m^{l}) + h \tag{23}$$

to keep I = 0

The implication of equation (23) is that when $m^{l} < 0$ (i. e. interest rates are falling) it will take shorter time for producers to start investing in inventories than it will in the case where m= constant, since

$$c'' \cdot \frac{dr}{dt} \leq c'm + h \quad (\text{ previous case })$$

will be violated later than equation (23), since

$$c'.\, m + h > h + c'(m + t\,m^{l})$$

In practical terms, this result implies that in cases where $m^{l}(t) < 0$, i. e. interest rates are falling, the time to start investing in inventories does not depend only on demand and current level of interest rates but also the rate of change of interest rates. Higher the rate of decline of interest rates, sooner will the producers start to invest in inventories, regardless of the level of interest rates.

The policy implication of this result however is interesting.

It states that faster the disinflationary policies are applied (i. e. faster decline in interest rates), sooner will the recovery of the economy start, even if the interest rates are very high.

Additionaly, equations (23) may be violated i. e. producers start to invest in inventories even if the demand is falling if the interest rates are falling faster.

Conclusions

High interest rates have profound negative impact on investment and production in that higher the interest rates longer will the investors wait to produce.

It is also true that if the interest rates are falling, the period of postponement will be shorter. More interestingly, it is also true that if the rate of decline of interest rates is very high, the postponement period for investment can be very short even if the current absolute level of interest rates are very high. An even more interesting case is when the rate of decline of interest rates are higher than the rate of decline of demand. In this case, the investors will begin to invest in inventories even if the demand for their commodity is declining.

It is, therefore, very clear that faster the application of disinflationary economic policies, sooner will the investment and the growth start.

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